7.7 Write and Apply Exponential and Power Functions

Before You wrote linear, quadratic, and other polynomial functions.

Now You will write exponential and power functions.

Why? So you can model biology problems, as in Example 5.

Key Vocabulary
- **power function,** *p. 428*
- **exponential function,** *p. 478*

In Chapter 2, you learned that two points determine a line. Similarly, two points determine an exponential curve.

EXAMPLE 1 Write an exponential function

Write an exponential function $y = ab^x$ whose graph passes through (1, 12) and (3, 108).

Solution

STEP 1 **Substitute** the coordinates of the two given points into $y = ab^x$.

$12 = ab^1$ **Substitute 12 for *y* and 1 for *x*.**

$108 = ab^3$ **Substitute 108 for *y* and 3 for *x*.**

STEP 2 **Solve** for a in the first equation to obtain $a = \frac{12}{b}$, and substitute this expression for a in the second equation.

$108 = \left(\frac{12}{b}\right)b^3$ **Substitute $\frac{12}{b}$ for *a* in second equation.**

$108 = 12b^2$ **Simplify.**

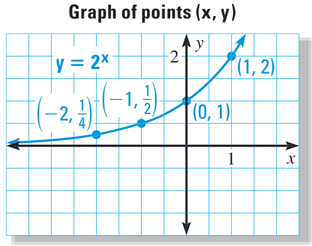
$9 = b^2$ **Divide each side by 12.**

$3 = b$ **Take the positive square root because $b > 0$.**

STEP 3 **Determine** that $a = \frac{12}{b} = \frac{12}{3} = 4$. So, $y = 4 \cdot 3^x$.

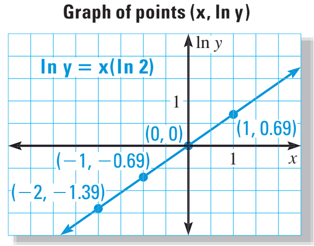
TRANSFORMING EXPONENTIAL DATA A set of more than two points (x, y) fits an exponential pattern if and only if the set of transformed points $(x, \ln y)$ fits a linear pattern.

Graph of points (x, y)



The graph is an exponential curve.

Graph of points (x, ln y)



The graph is a line.

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

1. Write an equation in slope-intercept form for the line through (2, 5) and (6, −3). **$y = -2x + 9$**
2. Write an equation in point-slope form for the line through (2, 4.53) and (5, 5.22). **$y - 4.53 = 0.23(x - 2)$ or $y - 5.22 = 0.23(x - 5)$**
3. What is the value of y if the point (10, y) is on the line $y = 5.8x + 2.4$? **60.4**

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 206–209.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3, p. 477

How do you determine whether a set of data fits an exponential pattern or a power pattern? **Tell students they will learn how to answer this question by using natural logarithms.**

NCTM STANDARDS

Standard 5: Use proper statistical methods to analyze data

Standard 6: Solve problems in math and other contexts

Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 73–74)
- Practice levels A, B, C (pp. 76–78)
- Study Guide (pp. 79–80)
- Catch-up for Absent Students (p. 81)
- Problem Solving Workshop (p. 82)
- Challenge (p. 83)

Workbooks
- Notetaking Guide (pp. 206–209)
- Practice Workbook (pp. 120–121)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the Algebra 2 Toolkit for more strategies for meeting individual needs.

Motivating the Lesson

Your family owns a small company. The company's revenue has been steadily increasing from year to year. You can use modeling techniques to determine whether a linear, exponential, or power function best models the relationship between years and revenue to predict future revenue.

3 TEACH

Extra Example 1

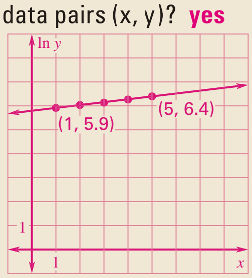
Write an exponential function $y = ab^x$ whose graph passes through (1, 10) and (4, 80). **$y = 5 \cdot 2^x$**

Extra Example 2

The table shows the number y of students enrolled in an elementary school during the xth year that the school has been open.

x	1	2	3	4	5
y	370	417	460	523	598

- Draw a scatter plot of the data pairs $(x, \ln y)$. Is an exponential model a good fit for the original data pairs (x, y)? **yes**



- Find an exponential model for the original data. **$y = 327(1.13)^x$ (Answers may vary slightly depending on the points used.)**

Extra Example 3

Use a graphing calculator to find an exponential model for the data in Extra Example 2. Predict the enrollment for the sixth year. **$y = 327(1.13)^x$; about 680 students**

EXAMPLE 2 Find an exponential model

SCOOTERS A store sells motor scooters. The table shows the number y of scooters sold during the xth year that the store has been open.

Year, x	1	2	3	4	5	6	7
Number of scooters sold, y	12	16	25	36	50	67	96

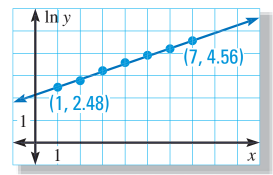
- Draw a scatter plot of the data pairs $(x, \ln y)$. Is an exponential model a good fit for the original data pairs (x, y)?
- Find an exponential model for the original data.

Solution

STEP 1 **Use** a calculator to create a table of data pairs $(x, \ln y)$.

x	1	2	3	4	5	6	7
$\ln y$	2.48	2.77	3.22	3.58	3.91	4.20	4.56

STEP 2 **Plot** the new points as shown. The points lie close to a line, so an exponential model should be a good fit for the original data.



STEP 3 **Find** an exponential model $y = ab^x$ by choosing two points on the line, such as (1, 2.48) and (7, 4.56). Use these points to write an equation of the line. Then solve for y.

$\ln y - 2.48 = 0.35(x - 1)$ **Equation of line**

$\ln y = 0.35x + 2.13$ **Simplify.**

$y = e^{0.35x + 2.13}$ **Exponentiate each side using base e.**

$y = e^{2.13}(e^{0.35})^x$ **Use properties of exponents.**

$y = 8.41(1.42)^x$ **Exponential model**

USE POINT-SLOPE FORM
Because the axes are x and $\ln y$, the point-slope form is rewritten as $\ln y - y_1 = m(x - x_1)$. The slope of the line through (1, 2.48) and (7, 4.56) is:

$$\frac{4.56 - 2.48}{7 - 1} \approx 0.35$$

EXPONENTIAL REGRESSION A graphing calculator that performs exponential regression uses all of the original data to find the best-fitting model.

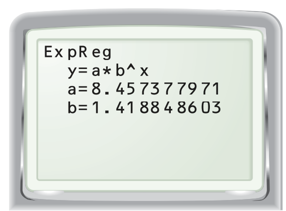
EXAMPLE 3 Use exponential regression

SCOOTERS Use a graphing calculator to find an exponential model for the data in Example 2. Predict the number of scooters sold in the eighth year.

Solution

Enter the original data into a graphing calculator and perform an exponential regression. The model is $y = 8.46(1.42)^x$.

Substituting $x = 8$ (for year 8) into the model gives $y = 8.46(1.42)^8 \approx 140$ scooters sold.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

Write an exponential function $y = ab^x$ whose graph passes through the given points.

1. (1, 6), (3, 24) $y = 3 \cdot 2^x$ **2.** (2, 8), (3, 32) $y = \frac{1}{2} \cdot 4^x$ **3.** (3, 8), (6, 64) $y = 2^x$

4. **WHAT IF?** In Examples 2 and 3, how would the exponential models change if the scooter sales were as shown in the table below?
The initial amount would change to 11.39 and the growth rate to 1.45.

Year, x	1	2	3	4	5	6	7
Number of scooters sold, y	15	23	40	52	80	105	140

WRITING POWER FUNCTIONS Recall from Lesson 6.3 that a power function has the form $y = ax^b$. Because there are only two constants (a and b), only two points are needed to determine a power curve through the points.

EXAMPLE 4 Write a power function

Write a power function $y = ax^b$ whose graph passes through (3, 2) and (6, 9).

Solution

STEP 1 **Substitute** the coordinates of the two given points into $y = ax^b$.

$2 = a \cdot 3^b$ **Substitute 2 for y and 3 for x.**

$9 = a \cdot 6^b$ **Substitute 9 for y and 6 for x.**

STEP 2 **Solve** for a in the first equation to obtain $a = \frac{2}{3^b}$, and substitute this expression for a in the second equation.

$9 = \left(\frac{2}{3^b}\right)6^b$ **Substitute $\frac{2}{3^b}$ for a in second equation.**

$9 = 2 \cdot 2^b$ **Simplify.**

$4.5 = 2^b$ **Divide each side by 2.**

$\log_2 4.5 = b$ **Take $\log_2$ of each side.**

$\frac{\log 4.5}{\log 2} = b$ **Change-of-base formula**

$2.17 \approx b$ **Use a calculator.**

STEP 3 **Determine** that $a = \frac{2}{3^{2.17}} \approx 0.184$. So, $y = 0.184x^{2.17}$.

✓ GUIDED PRACTICE for Example 4

Write a power function $y = ax^b$ whose graph passes through the given points.

5. (2, 1), (7, 6) $y = 0.371x^{1.43}$ **6.** (3, 4), (6, 15) $y = 0.492x^{1.91}$ **7.** (5, 8), (10, 34) $y = 0.278x^{2.09}$

8. **REASONING** Try using the method of Example 4 to find a power function whose graph passes through (3, 5) and (3, 7). What can you conclude?
Sample answer: The points cannot form a power function.

Extra Example 4

Write a power function $y = ax^b$ whose graph passes through (4, 6) and (8, 15). $y = 0.96x^{1.32}$

Key Questions to Ask for Example 4

- How does the method shown in the solution for Example 4 involve a system of equations? **The two given points are used in Step 1 to write a nonlinear system of two equations. In Steps 2 and 3, this system is solved by the substitution method.**
- Why is the change-of-base formula needed in Step 2 of the solution? **A calculator does not have a key for base 2 logarithms, so the change-of-base formula must be used to convert to either common or natural logarithms.**

Graphing Calculator

Students can also create the scatter plots for Examples 2 and 5 with their graphing calculators. If students are not familiar with this process, demonstrate how to enter the data into lists, use the STAT PLOT menu, select appropriate window settings, and plot the data points. Then demonstrate how to graph the equation of the model obtained in Step 2 of the solutions for these examples on the same screen as the scatter plot. When you discuss Examples 3 and 6, ask students to graph the equations obtained by exponential or power regression on the same screen as the scatter plot and observe if this gives a better fit than the equations obtained in Examples 2 and 5.